Measurements of Velocity Profile in Developing Liquid Flows

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An optical technique for the quantitative determination of point velocities in rapidly developing laminar flows of transparent liquids is useful as an experimental tool in problems of complex geometries and fluids. The accuracy of the technique has been confirmed by comparison of experimental data with the numerical solution of the equation of motion for the entrance region problem. A distributor capable of generating a flat velocity profile has been developed for this purpose.

The simpler laminar flow problems, that is, simple geometries and Newtonian fluids, may be scaled geometrically and dynamically so that the velocity gradients in real space as opposed to dimensionless space are minimized. Frequently however lack of knowledge of the scaling procedure makes this impossible. Consequently, a technique is required that will allow accurate determination of point velocities under arbitrary conditions.

The primary purpose of the work presented was to develop an experimental technique for such conditions. It is necessary to check any such technique against a known solution for a problem of developing flow. The most convenient problem for this purpose is flow in the entrance of a circular pipe, which, although not fully confirmed experimentally, may be assumed to have been satisfactorily solved by numerical techniques. In this problem the primary experimental requirement in addition to the measurement technique is to produce a known initial condition, which most conveniently is a flat velocity profile.

THEORY

The equation of motion, simplified by the Prandtl boundary-layer argument (*1*) as discussed by Kay (*2*) and Schlichting (*3*), may be expressed in dimensionless form (by use of cylindrical coordinates) for incompressible isothermal developing Newtonian flow in a circular pipe as

$$U\frac{\partial U}{\partial L} + V\frac{\partial U}{\partial R} = -\frac{dP}{dL} + \frac{1}{R}\frac{\partial U}{\partial R} + \frac{\partial^2 U}{\partial R^2} \qquad (1)$$

together with the continuity equation

$$\frac{\partial U}{\partial L} + \frac{1}{R}\frac{\partial (VR)}{\partial R} = 0 \qquad (2)$$

and restrictions

$$\text{I.C.} \qquad L = 0 \quad U = 1$$

$$\text{B.C. (1)} \qquad R = 0 \quad (dU/dR) = 0 \qquad (3)$$

$$\text{B.C. (2)} \qquad R = 1 \quad U = 0$$

where the dimensionless variables are given by

$$U = \frac{u}{\overline{u}}, \quad V = \frac{\rho v a}{\mu}, \quad R = \frac{r}{a}$$

$$L = \frac{l\mu}{a^2 \rho \overline{u}} = \frac{2l}{N_{Re}} \qquad (4)$$

$$P = \frac{p - p_o}{\rho\, \overline{u}^2}$$

Equations (1) to (3) pose a one-parameter problem in the sense that the dimensionless velocity may be expressed as a function of the dimensionless radial position with the dimensionless downstream distance the only parameter.

$$U = f_1(R, L) \qquad (5)$$

$$P = f_2(L) \qquad (6)$$

This problem has received much attention from a theoretical viewpoint with the approximate solutions of Boussinesq (*4*), Schiller (*5*), Langhaar (*6*), Campbell and Slattery (*7*), Goldstein (*8*), and the numerical solution of Hornbeck (*9*), confirmed by Christiansen and Lemmon (*10*).

Experimental investigations are due to Nikuradse (*11*), Reshotko (*12*), and Pfenninger (*13*). Nikuradse's work is reported as a series of smooth curves in Prandtl and Tietjens (*11*) and no details are given regarding the investigation. The others were primarily investigations into the stability of laminar flow at high Reynolds numbers prior to studies on boundary-layer suction. In these latter cases air was the experimental medium and converging entrance sections were used to generate a flat velocity profile. Reshotko used Reynolds numbers of 4,000 to 16,000 and Pfenninger of the order of 35,000; consequently the axial velocity gradients were minimal.

Figure 2 of reference *10* shows that only Pfenninger's data are in agreement with the numerical solution, proba-

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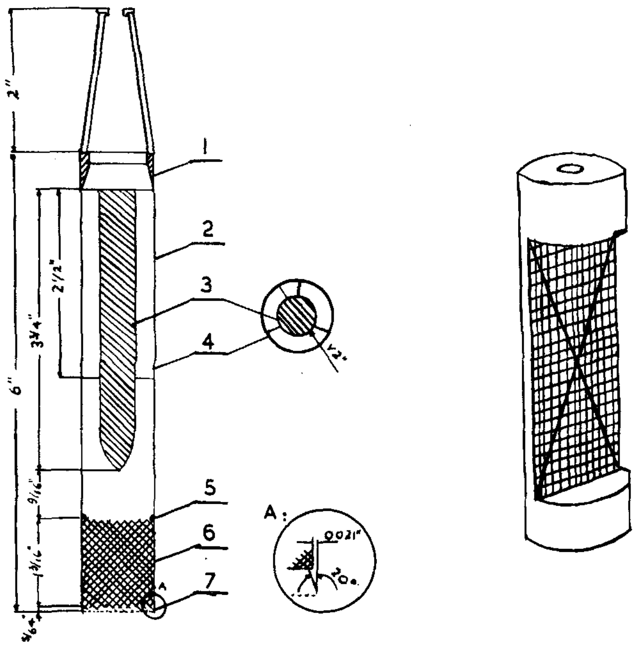

Fig. 1. Distributor and test scale. *1*, Brass sleeve with tripod; *2*, stainless steel tube; *3*, brass insert; *4*, brass guide; *5*, steel circlip; *6*, fifty-one brass gauzes (30 mesh); *7*, one brass gauze (60 mesh).

bly due to the impossibility of defining the location of the initial condition, when using (for stability at $N_{Re} >> 2{,}000$) an asymptotically converging duct, without resort either to mathematical methods or to measuring the profile in the entrance itself. Because of the extremely high Reynolds numbers used in Pfenninger's work, a very rough approximation to the location of the initial condition would be sufficient to yield the necessary accuracy.

DEVELOPMENT AND DESCRIPTION OF APPARATUS

Generation of a Flat Velocity Profile

A flat velocity profile may be produced by either a sudden contraction or a distributor. The latter is to be preferred when photographic techniques are employed. There are two reasons for this: the difficulty of measurement close to the tube entry, and the convenience of variable positioning of a distributor which permits the velocity measuring apparatus to remain in a fixed location. Furthermore, the effects of any residual liquid currents in the supply tank are minimized.

Producing a satisfactory distributor is difficult because of the tendency of the distributor to act as a filtration medium and to remove any suspended tracer particles with consequent blockage and nonuniformity of flow; also there is the difficulty of extending a flat profile to the walls of the tube.

Considerable preliminary work was required with various types of porous media but the most reliable, reproducible and simple method was found to be the introduction of a series of brass gauzes and an annular flow section in a demountable stainless steel cylinder.

The distributor (Figure 1) consisted of a 6-in. long stainless steel cylinder (wall thickness 0.04 in.) machined to fit the precision bore glass tube to within 0.0005 in., that is, interference fit. The lower end was bevelled to a knife edge on which was fixed a 60-mesh brass gauze. Above this were mounted fifty 30-mesh gauzes, each weave at 45 deg. to those adjacent. The coarser gauzes rested on a small projecting ring and were held in position by a steel circlip. Within the upper end of the cylinder a torpedo shaped brass insert (½-in. diameter) was located concentrically by means of three guides.

The arrangement allowed the number and size of gauzes to be varied relatively easily and it was possible to produce a profile that was flat within the limits of accuracy of the measuring technique.

Measurement of Point Velocities

General. In principle, point velocities may be measured by the insertion of pressure or velocity sensing devices (Pitot tubes or hot wire anemometers); however such instruments inevitably cause disturbance to the profile that they themselves are measuring. This situation is particularly unsatisfactory in the case of the more complex fluids. Consequently, *it is* necessary to adopt an optical technique using tracer particles despite the possible interference caused by the presence of rigid particles in the flowing fluid.

Optical Methods. Various optical methods for examining velocities have been described, in particular, by Fage and Townend (*14*), who demonstrated the presence of turbulence very close to the wall of a circular pipe. Their apparatus consisted of a microscope with a rotating objective. However, in order to measure the fluid velocities, continuous observation by the experimenter was required—an unsatisfactory procedure.

The same objection applies to the alternative means of reducing the apparent movement of particles to zero by means of a rotating glass cube positioned between the flowing fluid and the point of observation (a microscope).

The technique finally used is related to that described by Rushton (*15, 16*) and Taylor (*17*), in which particles were illuminated from the side, viewed against a dark ground, and photographed. The particles used had an extremely high reflectance coefficient, a density only slightly greater than that of water, and negligible settlement properties.

The illumination used was a high-intensity mercury vapor discharge lamp operated directly at mains frequency, providing illumination discontinuously at 100 flashes/sec. The path of each moving particle is thus seen as a discontinuous streak and the use of accurate intermittent illumination obviates the necessity of a calibrated shutter speed. As the paths of many particles are recorded simultaneously, a complete velocity profile may be derived from a single exposure.

Optical Arrangement. The glass tube was selected with due consideration to the quality of the interior polish. An area about 1.5 in. long and 0.025 in. deep on a plane along the tube axis was illuminated by the arrangement shown in Figure 2. This was achieved by means of a mercury vapor lamp (together with collimating lens) illuminating a 1-mm. slit, whose image was focused on to a second slit which was part of a box surrounding the experimental section of the tube.

In order to minimize refraction effects due to curvature at the air-glass-water surfaces, the tube was surrounded by a perspex box filled with water. The normal surfaces of the box were carefully aligned and light passing through the tube was reflected away by a surface set at 45 deg. to the direction of

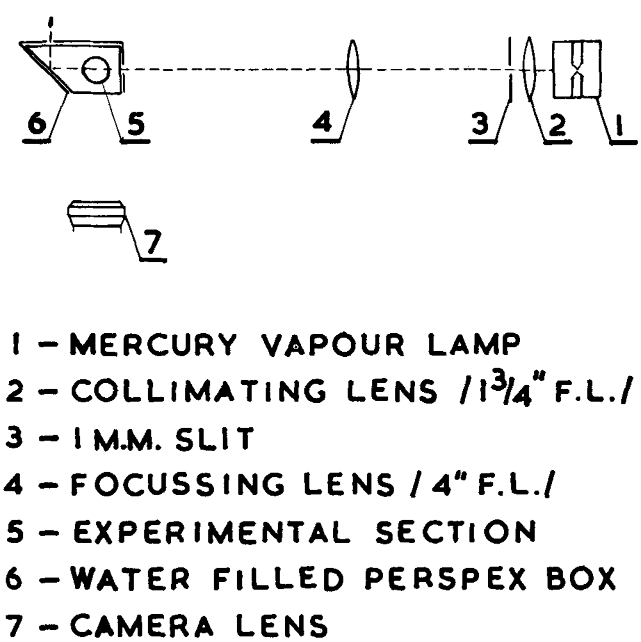

Fig. 2. Optical arrangement.

the incident light. The illuminated plane was viewed by a camera at right angles to the optical axis of the main light beam. All surfaces of the box were covered with black masking tape except for that facing the camera, and the slits where the main light beam entered and left the box.

Both standard (50 mm.) and telephoto (135 mm.) lenses were used with the camera (Edixa model B).

Film Measurement Apparatus

This consisted of a commercial unidirectional travelling microscope (capable of measurement to 0.01 mm.) mounted on a metal box containing provision for the illumination of 35-mm. film. The film itself was mounted as required on a table capable of angular adjustment so that it could be aligned accurately with the axis of the travelling microscope. The table was mounted such that it could be moved laterally by means of a micrometer screw. In this way the coordinates of any position on the film could be obtained with accuracy.

This method was found to be more accurate but less sensitive than measurements taken from a high-contrast enlarged print of the flow field. Both measurement techniques were used in the experimental program.

In working from prints an enlarger was set up and prints were made of a test grid exposure and flow field without adjusting the focus, enlargement, etc. A small check scale alongside the experimental section acted as a secondary length standard.

Description of Apparatus

General. The hydraulic recirculation system with provision for the necessary flow and temperature control is fully described by Kemblowski (*18*). A 48-in. length of precision bore glass tube (25.5-mm. diam.) of good optical quality was used as an experimental section. Coaxially with the glass tube an independently suspended rod was mounted on top of the apparatus for locating the distributor or test scale in any required position (Figure 3).

Tracer particles were added to the circulating liquid from a suspension and those of excessive size or agglomerates were removed by an 80-mesh filter.

The optical arrangements, including the camera, were housed in a curtained box to reduce background illumination; the camera itself was mounted independently to minimize vibration.

Test Grid (Figure 1). A test grid was prepared from a perspex rod to an interference fit. Over a 2-in. length the rod was cut to half its thickness so that the resulting plane coincided with the rod (and glass tube) axis. On this plane vertical and horizontal lines were machined at 0.025-in. intervals together with diagonals to mark the center of the plane.

EXPERIMENTAL PROCEDURE

The location of the illuminated area in the measuring section was determined by means of the test grid. When the optical bench and slit were properly adjusted the face of the grid was illuminated evenly. Tracer particles (Merlite, Mearl Corporation, New York) suspended in water were then added to the flow stream.

Subsequent replacement of the grid with the distributor positioned in the field of view yielded a check on the flow leaving the distributor (that is, absence of vortices, blockage, etc.). This was then located at the required position, relative to the illuminated area, the flow rate was adjusted, and photographs were taken of the flow field with Ilford H.P.3 or Agfa Isopan Record film. Exposure times were varied between 1/5 at f 2.8 and 4 sec. at f 8.

Each particle in the illuminated area appeared on the negative as a series of dashes, each dash representing the path of the particle during a single striking of the mercury arc (with an a.c. mains frequency of 50 cycles, this corresponds to a time interval of 0.01 sec.). The position of the particle relative to the tube wall could easily be determined as could the distance of travel for a given number of discharges. As necessary in rapid axial acceleration the distance travelled by a particle in a single interval of 0.01 sec. could be determined.

A check on the scaling of the negative (or point) was carried out by means of the scale inserted in the perspex box (Figure 2) for the axial direction and by use of the external tube diameter for the radial direction. No refraction effects were detected on photographs taken of the grid (Figure 1) in the tube.

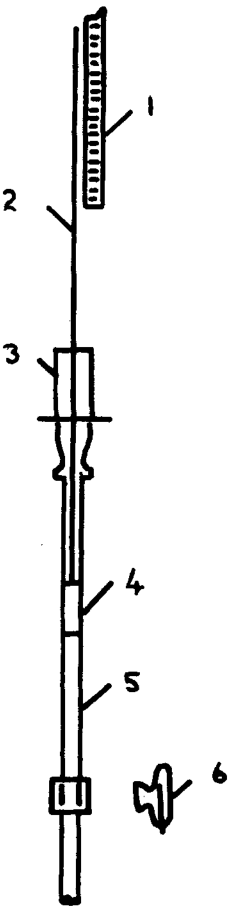

Fig. 3. Experimental arrangement. 1, scale; 2, adjusting rod; 3, filter; 4, distributors; 5, glass tube; 6, camera.

EXPERIMENTAL RESULTS

The measurements were made with water as an experimental medium. Results were obtained at a combination of downstream distances and Reynolds numbers.

These are tabulated accordingly:

Label on Figure 4	N_{Re}	Downstream distance l, mm.	Dimensionless downstream distance, L
a	1,485	16.5	0.00174
b	1,485	41.5	0.00438
c	1,485	91.5	0.00968
d	1,485	141.5	0.01492
e	1,505	150	0.0156
f	1,545	300	0.0304
g	987	300	0.0476
h	496	300	0.0948
i	498	900	0.2834

The data are plotted in Figure 4 in terms of Equation (5) and compared with the numerical solution (9).

The material in reference 18 has been deposited as document 9074 with the American Documentation Institute, Photoduplication Service, Library of Congress, Washington 25, D. C., and may be obtained for $1.25 for photoprints or 35-mm. microfilm.

DISCUSSION

Fully Developed Flow

Independent of the initial velocity profile, at large distances downstream equilibrium flow is attained. Consequently, the fully developed profile may be used to evaluate the potentiality of the measuring technique. From curve *i* in Figure 4 it can be seen that the data are in good agreement with the Poiseuille parabola.

Developing Flow

The most sensitive test of the efficiency of the distributor lies in the symmetry of the profiles. In Figure 4 velocities on opposing sides of the centre line are marked as being in regions A and B, respectively; the resulting symmetry is clearly satisfactory.

Curve *a* gives the profile at 16.5 mm. from the distributor in the region of the largest axial gradients. The agreement in the wall region is extremely good, in the central region, an increased scatter occurs which may reflect the efficiency of the distributor in producing a flat profile as much as the accuracy of the measuring technique. Curves (*b-h*) show the profiles at increasing downstream distances (decreasing axial gradients); again the agreement in the wall region is extremely good and that in the central region is within the limiting accuracy of the technique as demonstrated by curve *i*.

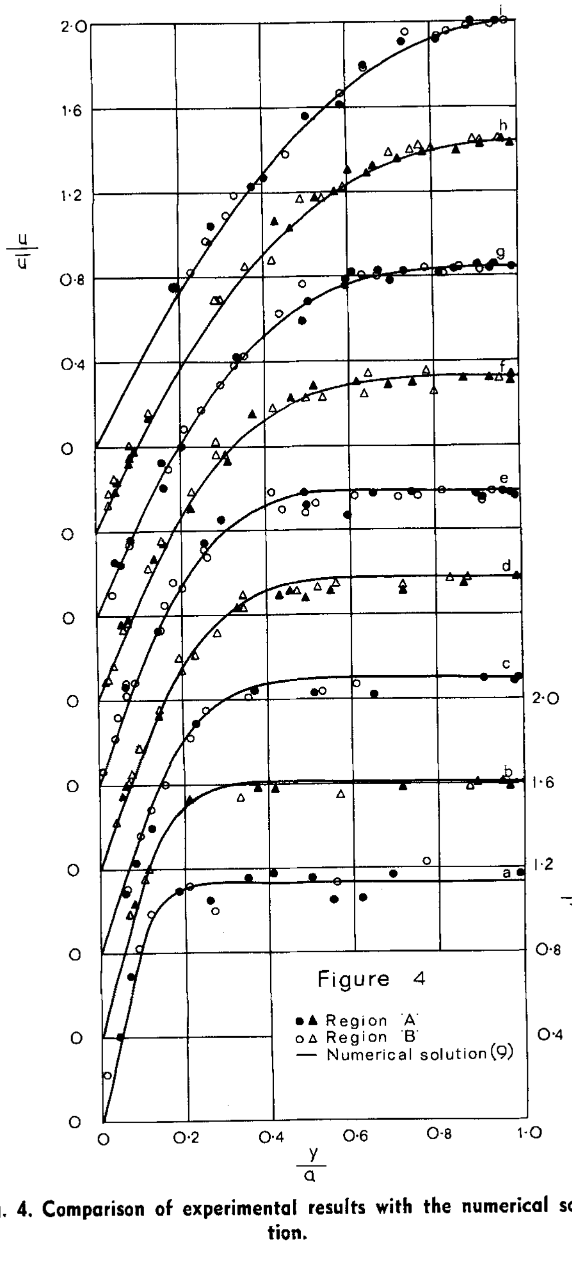

Fig. 4. Comparison of experimental results with the numerical solution.

CONCLUSIONS

An optical technique is presented which is capable of accurate determination of point velocities in transparent liquids for flow fields with large axial and transverse velocity gradients.

A distributor is described which is capable of generating a flat velocity profile.

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NOTATION

a	= radius
A, B	= opposite sides of center line in experimental section
l	= downstream distance (from the distributor)
L	= dimensionless downstream distance, $(l\,\mu/a^2\rho\bar{u})$
N_{Re}	= Reynolds number, $(2a\bar{u}\rho/\mu)$
P	= dimensionless pressure $(p - p_o/\rho\bar{u}^2)$
p	= pressure
p_o	= initial pressure
r	= radial position
R	= dimensionless radial position r/a
t	= temperature
u	= local axial velocity
$\bar{u}$	= average axial velocity
U	= dimensionless local axial velocity, $u/\bar{u}$
v	= radial velocity
V	= dimensionless radial velocity, $(\rho v a/\mu)$
y	= distance from wall
Y	= dimensionless distance from wall, $Y = 1 - R$
μ	= viscosity
ρ	= density

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